

Final state interaction and $B \rightarrow KK$ decays in perturbative QCD

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abstract

We predict branching ratios and CP asymmetries of the $B \rightarrow KK$ decays using perturbative QCD factorization theorem, in which tree, penguin, and annihilation contributions, including both factorizable and nonfactorizable ones, are expressed as convolutions of hard six-quark amplitudes with universal meson wave functions. The unitarity angle $\phi_3 = 90^\circ$ and the B and K meson wave functions extracted from experimental data of the $B \rightarrow K\pi$ and $\pi\pi$ decays are employed. Since the $B \rightarrow KK$ decays are sensitive to final-state-interaction effects, the comparison of our predictions with future data can test the neglect of these effects in the above formalism. The CP asymmetry in the $B^\pm \rightarrow K^\pm K^0$ modes and the $B_d^0 \rightarrow K^\pm K^\mp$ branching ratios depend on annihilation and nonfactorizable amplitudes. The $B \rightarrow KK$ data can also verify the evaluation of these contributions.

I. INTRODUCTION

The conventional approach to exclusive nonleptonic B meson decays relies on the factorization assumption (FA) [1], under which nonfactorizable and annihilation contributions are neglected and final-state-interaction (FSI) effects are assumed to be absent. Factorizable contributions are expressed as products of Wilson coefficients, meson decay constants, and hadronic transition form factors. Though analyses are simpler under this assumption, estimations of many important ingredients, such as tree and penguin (including electroweak penguin) contributions, and strong phases are not reliable. Moreover, the above naive FA suffers the problems of scale, infrared-cutoff and gauge dependences [2]. It is also difficult to explain the observed branching ratios of the $B \rightarrow J/\psi K^{(*)}$ decays in the FA approach, to which nonfactorizable and factorizable contributions are of the same order [3].

Perturbative QCD (PQCD) factorization theorem for exclusive heavy-meson decays has been developed some time ago [4–6], which goes beyond FA. PQCD is a method to separate hard components from a QCD process, which are treated by perturbation theory. Nonperturbative components are organized in the form of hadron wave functions, which can be extracted from experimental data. This prescription removes the infrared-cutoff dependence in PQCD. Since nonperturbative dynamics has been absorbed into wave functions, external quarks involved in hard amplitudes are on-shell, and gauge invariance of PQCD predictions is guaranteed. Contributions to hard parts from various topologies, such as tree, penguin and annihilation, including both factorizable and nonfactorizable contributions, can all be calculated. Without assuming FA, it is easy to achieve the scale independence in the PQCD approach.

Despite of the above merits of PQCD, an important subject, final state interaction (FSI), remains unsettled, which is nonperturbative but not universal. FSI effects in two-body decays have been assumed to be small. Though arguments and indications for this assumption have been supplied in [7], experimental justification is necessary. In this paper we shall propose to explore FSI effects by studying $B \rightarrow KK$ decays. It will be explained in Sec. II that these decays are more sensitive to FSI effects compared to $B \rightarrow K\pi$ and $\pi\pi$ decays. Employing the meson wave

functions and the unitarity angle $\phi_3 = 90^\circ$ determined in [7], we predict the branching ratios and the CP asymmetries of the $B^\pm \rightarrow K^\pm K^0$, $B_d^0 \rightarrow K^\pm K^\mp$ and $B_d^0 \rightarrow K^0 \bar{K}^0$ modes. The comparison of our predictions with future data can be used to estimate the importance of FSI effects. In particular, large observed $B_d^0 \rightarrow K^\pm K^\mp$ branching ratios and CP asymmetry in the $B_d^0 \rightarrow K^0 \bar{K}^0$ modes will imply strong FSI effects.

An essential difference between the FA and PQCD approaches is that annihilation and nonfactorizable amplitudes are neglected in the former, but calculable in the latter. It has been shown that annihilation contributions from the operator $O_{5,6}$ with the $(V-A)(V+A)$ structure, bypassing helicity suppression, are not negligible [7]. These contributions, being mainly imaginary, result in CP asymmetries in the $B \rightarrow \pi\pi$ decays, which are much larger than those predicted in FA [8,9]. Hence, measurements of CP asymmetries will distinguish the two approaches [9]. The $B^\pm \rightarrow K^\pm K^0$ modes contain both annihilation amplitudes from $O_{5,6}$ and nonfactorizable annihilation amplitudes from $O_{1,2}$, such that they exhibit substantial CP asymmetry. The branching ratios of the $B_d^0 \rightarrow K^\pm K^\mp$ modes, involving only nonfactorizable annihilation amplitudes, can not be estimated, or are vanishingly small in FA. The data of these two decays can verify PQCD evaluation of annihilation and nonfactorizable contributions.

FSI effects in the $B \rightarrow \pi\pi$, $K\pi$ and KK decays are compared in Sec. II. The PQCD formalism for annihilation and nonfactorizable contributions is reviewed in Sec. III. We present the factorization formulas of all the $B \rightarrow KK$ modes in Sec. IV, and perform a numerical analysis in Sec. V. Section VI is the conclusion.

II. FINAL STATE INTERACTION

FSI is a subtle and complicated subject. Most estimates of FSI effects in the literature [10] suffer ambiguities or difficulties. Kamal has pointed out that the enhancement of CP asymmetry in the $B^\pm \rightarrow K^0 \pi^\pm$ modes from order 0.5 % up to order (10-20)% [11] is due to an overestimation of FSI effects by a factor of 20 [12]. The smallness of FSI effects has been put forward by Bjorken [13] based on the color-transparency argument [14]. The renormalization-group (RG) analysis of soft gluon exchanges among initial- and final-state mesons [15] has also indicated that FSI effects are not important in two-body B meson decays. These discussions have led us to ignore FSI effects in the PQCD formalism. For example, the charge exchange in the rescattering $B^+ \rightarrow K^+ \pi^0 \rightarrow K^0 \pi^+$, regarded as occurring through short-distance quark-pair annihilation, is of higher order [7].

As stated in the Introduction, the neglect of FSI effects requires experimental justification. For this purpose, we propose to investigate the $B \rightarrow KK$ decays, which are more sensitive to FSI effects compared with the $B \rightarrow K\pi$ and $\pi\pi$ decays. Similar proposals have been presented in the literature [16,17] within the framework of $SU(3)$ symmetry. We make our argument explicit by means of the general expression for the $B \rightarrow \pi\pi$, $K\pi$ and KK decay amplitudes,

$$\mathcal{A} = V_u T + V_u P_u + V_c P_c + V_t P_t. \quad (1)$$

The factors $V_q = V_{qd} V_{qb}^*$, $q = u, c$, and t , are the products of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, T denotes the tree amplitude, and P_q denote the penguin amplitudes arising from internal q -quark contributions. FSI effects have been included in the amplitudes T and $P_{u,c,t}$.

Using the unitarity relation $V_c = -V_u - V_t$, Eq. (1) is rewritten as

$$\begin{aligned} \mathcal{A} &= V_u (T + P_u - P_c) + V_t (P_t - P_c), \\ &= V_u (T + P_u - P_c) \left[1 + \frac{V_t}{V_u} \frac{P_t - P_c}{T + P_u - P_c} \right], \\ &\equiv V_u (T + P_u - P_c) \left[1 + \frac{V_t}{V_u} \mathcal{R}_{\pi\pi(KK)} e^{i\delta_{\pi\pi(KK)}} \right], \end{aligned} \quad (2)$$

or

$$\begin{aligned} \mathcal{A} &= V_t (P_t - P_c) \left[1 + \frac{V_u}{V_t} \frac{T + P_u - P_c}{P_t - P_c} \right], \\ &\equiv V_t (P_t - P_c) \left[1 + \frac{V_u}{V_t} \mathcal{R}_{K\pi} e^{i\delta_{K\pi}} \right], \end{aligned} \quad (3)$$

where \mathcal{R} are the ratios of different amplitudes and δ the CP-conserving strong phases.

Without FSI, the various amplitudes T and $P_{u,c,t}$, namely, the ratios \mathcal{R} and the strong phases δ are calculable in PQCD. If FSI effects are important, they may change branching ratios or induce CP asymmetries of two-body B meson decays by varying \mathcal{R} and δ . For the $B \rightarrow \pi\pi$ decays, the ratio V_t/V_u is of order unity, but $\mathcal{R}_{\pi\pi}$ is small because of the large Wilson coefficients $a_1 = C_2 + C_1/N_c$ in T , N_c being number of colors. The exception is the $B_d^0 \rightarrow \pi^0 \pi^0$

mode, whose tree amplitude is proportional to the small Wilson coefficient $a_2 = C_1 + C_2/N_c$. The ratio $R_{K\pi}$ may be large, but its coefficient $V_u/V_t \sim R_b \lambda^2$, R_b and λ being the Wolfenstein parameters defined in Sec. IV, is small. Therefore, FSI effects in the $B \rightarrow \pi\pi$ and $K\pi$ decays are suppressed by $1/a_1$ and V_u/V_t , respectively. On the other hand, the $B \rightarrow \pi\pi$ and $K\pi$ decays have branching ratios of order 10^{-5} , which are larger than those of the $B \rightarrow KK$ decays (of order 10^{-6} as calculated in Sec. V). It has been also predicted in PQCD that the CP asymmetries in the $B \rightarrow \pi\pi$ and $K\pi$ decays are large: 30 – 40% in the former [8,9] and 10 – 15% in the latter [7]. These large values render FSI effects relatively mild.

For the $B \rightarrow KK$ decays, T arises only from small nonfactorizable annihilation diagrams for the $B^\pm \rightarrow K^\pm K^0$ and $B_d^0 \rightarrow K^\pm K^\mp$ modes, and vanishes for the $B_d^0 \rightarrow K^0 \bar{K}^0$ modes. Furthermore, V_t/V_u is of order unity. Hence, there is no suppression from the Wilson coefficients and from the CKM matrix elements, and FSI effects will be more significant. In the PQCD approach \mathcal{R}_{KK} is close to unity, corresponding to the branching ratios of order 10^{-6} for $B^\pm \rightarrow K^\pm K^0$ and $B_d^0 \rightarrow K^0 \bar{K}^0$, and 10^{-8} for $B_d^0 \rightarrow K^\pm K^\mp$. CP asymmetry vanishes in the $B_d^0 \rightarrow K^0 \bar{K}^0$ modes, because only the penguin operators contribute at leading order. However, if FSI contributes, the above results will be changed dramatically. For example, FSI effects could induce large T and $P_{u,c}$ via the rescattering of intermediate states DD and $\pi\pi$ produced from the tree operators, and P_t via the rescattering of intermediate states KK produced from the penguin operators. When \mathcal{R}_{KK} deviates from unity through rescattering processes, the branching ratios and CP asymmetries of the $B \rightarrow KK$ decays could be enhanced.

We show how FSI effects modify amplitudes of various topologies in the $B \rightarrow KK$ decays in Table I. For more allowed intermediate states, refer to [17]. It is obvious that the rescattering processes $DD(\pi\pi) \rightarrow KK$ may be important due to the large $B \rightarrow DD(\pi\pi)$ branching ratios. For example, $B(B_d^0 \rightarrow \pi^\pm \pi^\pm)$ is of order 10^{-5} . It is then possible that FSI effects could be significant enough to increase $B(B_d^0 \rightarrow K^\pm K^\mp)$ from order 10^{-8} to above 10^{-7} . For a similar reason, the rescattering processes could induce large $P_{u,c}$ with the CKM matrix elements $V_{u,c}$, which, as interfered with the penguin contributions, result in sizable CP asymmetry in the $B_d^0 \rightarrow K^0 \bar{K}^0$ modes. Hence, large CP asymmetry observed in the $B_d^0 \rightarrow K^0 \bar{K}^0$ modes and large deviation of the observed $B_d^0 \rightarrow K^\pm K^\mp$ branching ratios from the PQCD predictions will indicate strong FSI effects.

III. NONFACTORIZABLE AND ANNIHILATION CONTRIBUTIONS

PQCD factorization theorem for exclusive nonleptonic B meson decays has been briefly reviewed in [7]. In this section we simply sketch the idea of PQCD factorization theorem, concentrating on its application to nonfactorizable and annihilation amplitudes in the $B \rightarrow KK$ decays.

In perturbation theory nonperturbative dynamics is reflected by infrared divergences in radiative corrections. These infrared divergences can be separated and absorbed into a B meson wave function or a kaon wave function order by order [4]. A formal definition of the meson wave functions as matrix elements of nonlocal operators can be constructed, which, if evaluated perturbatively, reproduces the infrared divergences. Certainly, one can not derive a wave function in perturbation theory, but parametrizes it as a parton model, which describes how a parton (valence quark, if a leading-twist wave function is referred) shares meson momentum. The meson wave functions, characterized by the QCD scale Λ_{QCD} , must be determined by nonperturbative means, such as lattice gauge theory and QCD sum rules, or extracted from experimental data. In the application below small parton transverse momenta k_T are included, and the characteristic scale is replaced by $1/b$ with b being a variable conjugate to k_T .

After absorbing infrared divergences into the meson wave functions, the remaining part of radiative corrections is infrared finite. This part can be evaluated perturbatively as a hard amplitude with six on-shell external quarks, four of which correspond to the four-fermion operators and two of which are the spectator quarks of the B or K mesons. Note that the b quark carries various momenta, whose distribution is described by the B meson wave function introduced above. The six-quark amplitude contains all possible Feynman diagrams, which include both factorizable and nonfactorizable tree, penguin and annihilation contributions. A factorizable diagram involves hard gluon exchanges among valence quarks of the B meson or of a kaon. A nonfactorizable diagram involves hard gluon exchanges between the valence quarks of different mesons. That is, the PQCD formalism does not rely on FA.

The hard amplitude is characterized by the virtuality t of involved internal particles, which is of order M_B , and by the W boson mass M_W . The hard scale t reflects the specific dynamics of a decay mode, while M_W serves the scale, at which the matching conditions of the effective weak Hamiltonian to the full Hamiltonian are defined. The study of the pion form factor has indicated that the choice of t as the maximum of internal particle virtualities minimizes next-to-leading-order corrections to hard amplitudes [18]. Large logarithmic corrections are organized by RG methods. The results consist of the evolution from M_W down to t described by the Wilson coefficients, the evolution from t to $1/b$, and a Sudakov factor. The two evolutions are governed by different anomalous dimensions, since loop corrections associated with spectator quarks contribute, when the energy scale runs to below t . The Sudakov factor suppresses

the long-distance contributions from the large b region, and vanishes as $b = 1/\Lambda_{\text{QCD}}$. This suppression guarantees the applicability of PQCD to exclusive decays around the energy scale of the B meson mass [4].

A salient feature of PQCD factorization theorem is the universality of nonperturbative wave functions. Because of universality, meson wave functions extracted from some decay modes can be employed to make predictions for other modes. We have determined the B and K meson wave functions from the experimental data of the $B \rightarrow K\pi$ and $\pi\pi$ decays [7], and the unitarity angle $\phi_3 = 90^\circ$ from the CLEO data of the ratio [19],

$$R = \frac{B(B_d^0 \rightarrow K^\pm \pi^\mp)}{B(B^\pm \rightarrow K^0 \pi^\pm)} = 0.95 \pm 0.30, \quad (4)$$

where $B(B_d^0 \rightarrow K^\pm \pi^\mp)$ represents the CP average of the branching ratios $B(B_d^0 \rightarrow K^+ \pi^-)$ and $B(\bar{B}_d^0 \rightarrow K^- \pi^+)$. It has been emphasized that the $B \rightarrow \pi\pi$ data can be explained using the same angle $\phi_3 = 90^\circ$ in PQCD, contrary to the conclusion in [20,21], where an angle larger than 100° must be adopted. In this work we shall predict the branching ratios and CP asymmetries of the $B \rightarrow KK$ decays in the PQCD formalism employing the above meson wave functions and the unitarity angle.

Factorizable annihilation contributions correspond to the time-like kaon form factor. It is known that annihilation contributions from the O_{1-4} operators with the $(V-A)(V-A)$ structure vanish because of helicity suppression. However, those from the $O_{5,6}$ operators with the $(V-A)(V+A)$ structure bypass helicity suppression, and turn out to be comparable with penguin contributions [7]. Without FSI in PQCD, strong phases arise from non-pinch singularities of quark and gluon propagators in annihilation and nonfactorizable diagrams. Especially, annihilation amplitudes are the main source of strong phases [7]. In the FA and Beneke-Buchalla-Neubert-Sachrajda (BBNS) [22,23] approaches, where annihilation diagrams are not taken into account, strong phases come from the Bander-Silverman-Soni (BSS) mechanism [24] and from the extraction of the scale dependence from hadronic matrix elements [25]. As shown in [9], these sources are in fact next-to-leading-order. As a consequence, CP asymmetries predicted in FA and BBNS are smaller than those predicted in PQCD. Nonfactorizable amplitudes have been also considered in the BBNS approach, which are, however, treated in a different way. For example, they are real because of some approximation in [22], but complex in PQCD [26]. Generally speaking, nonfactorizable contributions are less important compared to factorizable ones except in the cases where factorizable contributions are proportional to the small Wilson coefficients a_2 or vanish.

As stated before, the $B^\pm \rightarrow K^\pm K^0$ decays involve both annihilation amplitudes from $O_{5,6}$ and nonfactorizable annihilation amplitudes from $O_{1,2}$. Their interference then leads to substantial CP asymmetry in PQCD. The $B_d^0 \rightarrow K^\pm K^\mp$ decays involve only nonfactorizable annihilation amplitudes from tree and penguin operators, such that their branching ratios can not be estimated, or are vanishingly small in the FA and BBNS approaches. These quantities mark the essential differences among FA, BBNS and PQCD. The comparison of our predictions for the CP asymmetry in the $B^\pm \rightarrow K^\pm K^0$ decays and for the $B_d^0 \rightarrow K^\pm K^\mp$ branching ratios with future data will justify our evaluation of annihilation and nonfactorizable contributions, and distinguish the FA, BBNS and PQCD approaches.

IV. FACTORIZATION FORMULAS

We present the factorization formulas of the $B \rightarrow KK$ decays in this section. The effective Hamiltonian for the flavor-changing $b \rightarrow d$ transition is given by [27]

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_q \left[C_1(\mu) O_1^{(q)}(\mu) + C_2(\mu) O_2^{(q)}(\mu) + \sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right], \quad (5)$$

with the CKM matrix elements $V_q = V_{qd}^* V_{qb}$ and the operators

$$\begin{aligned} O_1^{(q)} &= (\bar{d}_i q_j)_{V-A} (\bar{q}_j b_i)_{V-A}, & O_2^{(q)} &= (\bar{d}_i q_i)_{V-A} (\bar{q}_j b_j)_{V-A}, \\ O_3 &= (\bar{d}_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V-A}, & O_4 &= (\bar{d}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A}, \\ O_5 &= (\bar{d}_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V+A}, & O_6 &= (\bar{d}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A}, \\ O_7 &= \frac{3}{2} (\bar{d}_i b_i)_{V-A} \sum_q e_q (\bar{q}_j q_j)_{V+A}, & O_8 &= \frac{3}{2} (\bar{d}_i b_j)_{V-A} \sum_q e_q (\bar{q}_j q_i)_{V+A}, \\ O_9 &= \frac{3}{2} (\bar{d}_i b_i)_{V-A} \sum_q e_q (\bar{q}_j q_j)_{V-A}, & O_{10} &= \frac{3}{2} (\bar{d}_i b_j)_{V-A} \sum_q e_q (\bar{q}_j q_i)_{V-A}, \end{aligned} \quad (6)$$

i and j being the color indices. Using the unitarity condition, the CKM matrix elements for the penguin operators O_{3-10} can also be expressed as $V_u + V_c = -V_t$. The unitarity angle ϕ_3 is defined via

$$V_{ub} = |V_{ub}| \exp(-i\phi_3) . \quad (7)$$

Adopting the Wolfenstein parametrization for the CKM matrix upto $\mathcal{O}(\lambda^3)$,

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} , \quad (8)$$

we have the parameters [28]

$$\begin{aligned} \lambda &= 0.2196 \pm 0.0023 , \\ A &= 0.819 \pm 0.035 , \\ R_b &\equiv \sqrt{\rho^2 + \eta^2} = 0.41 \pm 0.07 . \end{aligned} \quad (9)$$

For the $B^\pm \rightarrow K^\pm K^0$ decays, the operators $O_{1,2}^{(u)}$ contribute via the annihilation topology, in which the fermion flow forms two loops as shown in Fig. 1. $O_{1,2}^{(c)}$ do not contribute at leading order of α_s . O_{3-10} contribute via the penguin topology with the light quark $q = s$ and via the annihilation topology with $q = u$, in which the fermion flow forms one loop. As evaluating hard amplitudes, an additional minus sign should be associated with the $O_{1,2}^{(u)}$ contributions, that contain two fermion loops. $O_{1,3,5,7,9}$ give both factorizable and nonfactorizable (color-suppressed) contributions, while $O_{2,4,6,8,10}$ give only factorizable ones because of the color flow. The electroweak penguin contributions from O_{7-10} have been included in the same way as those from the QCD penguins O_{3-6} . Obviously, the electroweak penguins are less important because of the small electromagnetic coupling.

The diagrams for the $B_d^0 \rightarrow K^\pm K^\mp$ decays are displayed in Fig. 2. The operators $O_{1,2}^{(u)}$ contribute via the annihilation topology, which contain one fermion loop. $O_{1,2}^{(c)}$ do not contribute at leading order of α_s . O_{3-10} contribute via the annihilation topology with the light quark $q = s$ or u , in which the fermion flow forms two loops. In these modes $O_{2,4,6,8,10}$ give both factorizable and nonfactorizable contributions, while $O_{1,3,5,7,9}$ give only factorizable ones because of the color flow. Only the operators O_{3-10} contribute to the $B_d^0 \rightarrow K^0 \bar{K}^0$ modes via the penguin topology with the light quark $q = s$ and via the annihilation topology with the light quark $q = s$ or d . The penguin contributions contain one fermion loop. The $q = s$ annihilation amplitudes involve two fermion loops, while the $q = d$ annihilation amplitudes contain both cases of one fermion loop and of two fermion loops as shown in Fig. 3.

The B meson momentum in light-cone coordinates is chosen as $P_1 = (M_B/\sqrt{2})(1, 1, \mathbf{0}_T)$. Momenta of the two kaons are chosen as $P_2 = (M_B/\sqrt{2})(1, 0, \mathbf{0}_T)$ and $P_3 = P_1 - P_2$. We shall drop the contributions of order $(M_K/M_B)^2 \sim 5\%$, M_K being the kaon mass. The B meson is at rest with the above parametrization of momenta. We define the momenta of light valence quark in the B meson as k_1 , where k_1 has a plus component k_1^+ , giving the momentum fraction $x_1 = k_1^+/P_1^+$, and small transverse components \mathbf{k}_{1T} . The two light valence quarks in the kaon involved in the $B \rightarrow K$ transition form factor carry the longitudinal momenta $x_2 P_2$ and $(1 - x_2)P_2$, and small transverse momenta \mathbf{k}_{2T} and $-\mathbf{k}_{2T}$, respectively. The two light valence quarks in the other kaon carry the longitudinal momenta $x_3 P_3$ and $(1 - x_3)P_3$, and small transverse momenta \mathbf{k}_{3T} and $-\mathbf{k}_{3T}$, respectively.

The Sudakov resummations of large logarithmic corrections to the B and K meson wave functions lead to the exponentials $\exp(-S_B)$, $\exp(-S_{K_2})$ and $\exp(-S_{K_3})$, respectively, with the exponents

$$\begin{aligned} S_B(t) &= s(x_1 P_1^+, b_1) + 2 \int_{1/b_1}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})) , \\ S_{K_2}(t) &= s(x_2 P_2^+, b_2) + s((1 - x_2)P_2^+, b_2) + 2 \int_{1/b_2}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})) , \\ S_{K_3}(t) &= s(x_3 P_3^-, b_3) + s((1 - x_3)P_3^-, b_3) + 2 \int_{1/b_3}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})) . \end{aligned} \quad (10)$$

The variable b_1 , b_2 , and b_3 conjugate to the parton transverse momentum k_{1T} , k_{2T} , and k_{3T} represents the transverse extent of the B and K mesons, respectively. The exponent s is written as [29–31]

$$s(Q, b) = \int_{1/b}^Q \frac{d\mu}{\mu} \left[\ln \left(\frac{Q}{\mu} \right) A(\alpha_s(\mu)) + B(\alpha_s(\mu)) \right] , \quad (11)$$

where the anomalous dimensions A to two loops and B to one loop are

$$A = C_F \frac{\alpha_s}{\pi} + \left[\frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27}f + \frac{2}{3}\beta_0 \ln \left(\frac{e^{\gamma_E}}{2} \right) \right] \left(\frac{\alpha_s}{\pi} \right)^2 ,$$

$$B = \frac{2}{3} \frac{\alpha_s}{\pi} \ln \left(\frac{e^{2\gamma_E-1}}{2} \right) , \quad (12)$$

with $C_F = 4/3$ a color factor, $f = 4$ the active flavor number, and γ_E the Euler constant. The one-loop expression of the running coupling constant,

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda_{\text{QCD}}^2)} , \quad (13)$$

is substituted into Eq. (11) with the coefficient $\beta_0 = (33 - 2f)/3$. The anomalous dimension $\gamma = -\alpha_s/\pi$ describes the RG evolution from t to $1/b$.

The decay rates of $B^\pm \rightarrow K^\pm K^0$ have the expressions

$$\Gamma = \frac{G_F^2 M_B^3}{128\pi} |\mathcal{A}|^2 . \quad (14)$$

The decay amplitudes \mathcal{A}^+ and \mathcal{A}^- corresponding to $B^+ \rightarrow K^+ K^0$ and $B^- \rightarrow K^- K^0$, respectively, are written as

$$\mathcal{A}^+ = f_K V_t^* F_{46}^{P(s)} + V_t^* \mathcal{M}_{46}^{P(s)} + f_B V_t^* F_{a6}^{P(u)} + V_t^* \mathcal{M}_{a46}^{P(u)} - V_u^* \mathcal{M}_{a1} , \quad (15)$$

$$\mathcal{A}^- = f_K V_t F_{46}^{P(s)} + V_t \mathcal{M}_{46}^{P(s)} + f_B V_t F_{a6}^{P(u)} + V_t \mathcal{M}_{a46}^{P(u)} - V_u \mathcal{M}_{a1} , \quad (16)$$

with the kaon decay constant f_K . The notation $F(\mathcal{M})$ represents factorizable (nonfactorizable) contributions, where the indices a and $P(q)$ denote the annihilation and penguin topologies, respectively, with the q quark pair emitted from the electroweak penguins, and the subscripts 1, 4 and 6 label the Wilson coefficients appearing in the factorization formulas. The nonfactorizable amplitude \mathcal{M}_{a1} are from the operators $O_{1,2}^{(u)}$.

The decay rates of $B_d^0 \rightarrow K^\pm K^\mp$ have the similar expressions with the amplitudes

$$\mathcal{A} = V_t^* (\mathcal{M}_{a35}^{P(u)} + \mathcal{M}_{a35}^{P(s)}) - V_u^* \mathcal{M}_{a2} , \quad (17)$$

$$\bar{\mathcal{A}} = V_t (\mathcal{M}_{a35}^{P(u)} + \mathcal{M}_{a35}^{P(s)}) - V_u \mathcal{M}_{a2} , \quad (18)$$

for $B_d^0 \rightarrow K^+ K^-$ and $\bar{B}_d^0 \rightarrow K^- K^+$, respectively. The notations are similar to those in Eqs. (15) and (16). The decay amplitudes for $B_d^0 \rightarrow K^0 \bar{K}^0$ and $\bar{B}_d^0 \rightarrow K^0 \bar{K}^0$ are written as

$$\mathcal{A}' = f_K V_t^* F_{46}^{P(s)} + V_t^* \mathcal{M}_{46}^{P(s)} + f_B V_t^* F_{a6}^{P(d)} + V_t^* (\mathcal{M}_{a46}^{P(d)} + \mathcal{M}_{a35}^{P(d)} + \mathcal{M}_{a35}^{P(s)}) , \quad (19)$$

$$\bar{\mathcal{A}}' = f_K V_t F_{46}^{P(s)} + V_t \mathcal{M}_{46}^{P(s)} + f_B V_t F_{a6}^{P(d)} + V_t (\mathcal{M}_{a46}^{P(d)} + \mathcal{M}_{a35}^{P(d)} + \mathcal{M}_{a35}^{P(s)}) , \quad (20)$$

respectively.

The factorizable contributions are written as

$$F_{46}^{P(s)} = F_4^{P(s)} + F_6^{P(s)} ,$$

$$F_4^{P(s)} = 16\pi C_F M_B^2 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_B(x_1, b_1)$$

$$\times \{ [(1+x_3)\phi_K(x_3) + r_K(1-2x_3)\phi'_K(x_3)] E_{e4}^{(s)}(t_e^{(1)}) h_e(x_1, x_3, b_1, b_3)$$

$$+ 2r_K \phi'_K(x_3) E_{e4}^{(s)}(t_e^{(2)}) h_e(x_3, x_1, b_3, b_1) \} , \quad (21)$$

$$F_6^{P(s)} = 32\pi C_F M_B^2 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_B(x_1, b_1)$$

$$\times r_K \{ [\phi_K(x_3) + r_K(2+x_3)\phi'_K(x_3)] E_{e6}^{(s)}(t_e^{(1)}) h_e(x_1, x_3, b_1, b_3)$$

$$+ [x_1 \phi_K(x_3) + 2r_K(1-x_1)\phi'_K(x_3)] E_{e6}^{(s)}(t_e^{(2)}) h_e(x_3, x_1, b_3, b_1) \} , \quad (22)$$

$$F_{a6}^{P(q)} = 32\pi C_F M_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3$$

$$\times r_K \{ [x_3 \phi_K(1-x_2)\phi'_K(1-x_3) + 2\phi'_K(1-x_2)\phi_K(1-x_3)] E_{a6}^{(q)}(t_a^{(1)}) h_a(x_2, x_3, b_2, b_3)$$

$$+ [2\phi_K(1-x_2)\phi'_K(1-x_3) + x_2 \phi'_K(1-x_2)\phi_K(1-x_3)] E_{a6}^{(q)}(t_a^{(2)}) h_a(x_3, x_2, b_3, b_2) \} , \quad (23)$$

for the light quarks $q = u$ and d . The evolution factors are given by

$$E_{ei}^{(s)}(t) = \alpha_s(t) a_i^{(s)}(t) \exp[-S_B(t) - S_{K3}(t)] , \quad (24)$$

$$E_{ai}^{(q)}(t) = \alpha_s(t) a_i^{(q)}(t) \exp[-S_{K2}(t) - S_{K3}(t)] . \quad (25)$$

Notice the arguments $1 - x_2$ and $1 - x_3$ of the kaon wave functions ϕ_K and ϕ'_K in Eqs. (26) and (23). The explicit expressions of the kaon wave functions will be given in Sec. V, where x represents the momentum fraction of the light u or d quark. However, to render the annihilation contributions for $q = s$ and for $q = u$ or d have the same hard parts, we have labelled the s quark momentum by x in the latter case, and changed the arguments of the kaon wave functions to $1 - x$.

The factorizable annihilation contribution associated with the Wilson coefficient $a_4^{(q)}$ from Fig. 1(c) is identical to zero because of helicity suppression as indicated by

$$\begin{aligned} F_{a4}^{P(q)} = & 16\pi C_F M_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \\ & \times \{ [-x_3 \phi_K(1 - x_2) \phi_K(1 - x_3) - 2r_K^2(1 + x_3) \phi'_K(1 - x_2) \phi'_K(1 - x_3)] \\ & \times E_{a4}^{(q)}(t_a^{(1)}) h_a(x_2, x_3, b_2, b_3) \\ & + [x_2 \phi_K(1 - x_2) \phi_K(1 - x_3) + 2r_K^2(1 + x_2) \phi'_K(1 - x_2) \phi'_K(1 - x_3)] \\ & \times E_{a4}^{(q)}(t_a^{(2)}) h_a(x_3, x_2, b_3, b_2) \} . \end{aligned} \quad (26)$$

The helicity suppression does not apply to the annihilation contributions associated with $a_6^{(q)}$, and the two terms in Eq. (23) are constructive. It is easy to confirm these observations by interchanging the integration variables x_2 and x_3 in the second terms of Eqs. (23) and (26). The factorization formulas for F_{a1} from Fig. 1(a) and for F_{a2} from Fig. 2(a), associated with the Wilson coefficient $a_1(t_a)$ and $a_2(t_a)$, respectively, are the same as $F_{a4}^{P(q)}$, *i.e.*, vanish. The expressions of $F_{a35}^{P(q)}$ from Figs. 2(b), 2(c), 3(c), and 3(d), associated with the Wilson coefficients $a_3^{(q)}(t_a) + a_5^{(q)}(t_a)$, are also the same as $F_{a4}^{P(q)}$ and vanish.

The hard functions h 's in Eqs (21)-(23), are given by

$$\begin{aligned} h_e(x_1, x_3, b_1, b_3) = & K_0(\sqrt{x_1 x_3} M_B b_1) \\ & \times [\theta(b_1 - b_3) K_0(\sqrt{x_3} M_B b_1) I_0(\sqrt{x_3} M_B b_3) \\ & + \theta(b_3 - b_1) K_0(\sqrt{x_3} M_B b_3) I_0(\sqrt{x_3} M_B b_1)] , \end{aligned} \quad (27)$$

$$\begin{aligned} h_a(x_2, x_3, b_2, b_3) = & \left(\frac{i\pi}{2} \right)^2 H_0^{(1)}(\sqrt{x_2 x_3} M_B b_2) \\ & \times \left[\theta(b_2 - b_3) H_0^{(1)}(\sqrt{x_3} M_B b_2) J_0(\sqrt{x_3} M_B b_3) \right. \\ & \left. + \theta(b_3 - b_2) H_0^{(1)}(\sqrt{x_3} M_B b_3) J_0(\sqrt{x_3} M_B b_2) \right] . \end{aligned} \quad (28)$$

The derivation of h , from the Fourier transformation of the lowest-order H , is similar to that for the $B \rightarrow D\pi$ decays [3,26]. The hard scales t are chosen as the maxima of the virtualities of internal particles involved in b quark decay amplitudes, including $1/b_i$:

$$\begin{aligned} t_e^{(1)} &= \max(\sqrt{x_3} M_B, 1/b_1, 1/b_3) , \\ t_e^{(2)} &= \max(\sqrt{x_1} M_B, 1/b_1, 1/b_3) , \\ t_a^{(1)} &= \max(\sqrt{x_3} M_B, 1/b_2, 1/b_3) , \\ t_a^{(2)} &= \max(\sqrt{x_2} M_B, 1/b_2, 1/b_3) , \end{aligned} \quad (29)$$

which decrease higher-order corrections. The Sudakov factor in Eq. (10) suppresses long-distance contributions from the large b (*i.e.*, large $\alpha_s(t)$) region, and improves the applicability of PQCD to B meson decays.

For the nonfactorizable amplitudes, the factorization formulas involve the kinematic variables of all the three mesons, and the Sudakov exponent is given by $S = S_B + S_{K2} + S_{K3}$. The integration over b_3 can be performed trivially, leading to $b_3 = b_1$ or $b_3 = b_2$. Their expressions are

$$\begin{aligned}
\mathcal{M}_{46}^{P(s)} &= \mathcal{M}_4^{P(s)} + \mathcal{M}_6^{P(s)} , \\
\mathcal{M}_4^{P(s)} &= -32\pi C_F \sqrt{2N_c} M_B^2 \int_0^1 [dx] \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \phi_K(x_2) \\
&\quad \times \{ [(x_1 - x_2) \phi_K(x_3) + r_K x_3 \phi'_K(x_3)] E_{e4}^{(s)'}(t_d^{(1)}) h_d^{(1)}(x_1, x_2, x_3, b_1, b_2, b_1) \\
&\quad + [(1 - x_1 - x_2 + x_3) \phi_K(x_3) - r_K x_3 \phi'_K(x_3)] E_{e4}^{(s)'}(t_d^{(2)}) h_d^{(2)}(x_1, x_2, x_3, b_1, b_2, b_1) \} , \tag{30}
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_6^{P(s)} &= -32\pi C_F \sqrt{2N_c} M_B^2 \int_0^1 [dx] \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \phi'_K(x_2) \\
&\quad \times r_K \{ [(x_1 - x_2) \phi_K(x_3) + r_K (x_1 - x_2 - x_3) \phi'_K(x_3)] \\
&\quad \times E_{e6}^{(s)'}(t_d^{(1)}) h_d^{(1)}(x_1, x_2, x_3, b_1, b_2, b_1) \\
&\quad + [(1 - x_1 - x_2) \phi_K(x_3) + r_K (1 - x_1 - x_2 + x_3) \phi'_K(x_3)] \\
&\quad \times E_{e6}^{(s)'}(t_d^{(2)}) h_d^{(2)}(x_1, x_2, x_3, b_1, b_2, b_1) \} , \tag{31}
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_{a46}^{P(q)} &= \mathcal{M}_{a4}^{P(q)} + \mathcal{M}_{a6}^{P(q)} , \\
\mathcal{M}_{a4}^{P(q)} &= 32\pi C_F \sqrt{2N_c} M_B^2 \int_0^1 [dx] \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \\
&\quad \times \{ [x_3 \phi_K(1 - x_2) \phi_K(1 - x_3) - r_K^2 (x_1 - x_2 - x_3) \phi'_K(1 - x_2) \phi'_K(1 - x_3)] \\
&\quad \times E_{a4}^{(q)'}(t_f^{(1)}) h_f^{(1)}(x_1, x_2, x_3, b_1, b_2, b_2) \\
&\quad - [(x_1 + x_2) \phi_K(1 - x_2) \phi_K(1 - x_3) + r_K^2 (2 + x_1 + x_2 + x_3) \phi'_K(1 - x_2) \phi'_K(1 - x_3)] \\
&\quad \times E_{a4}^{(q)'}(t_f^{(2)}) h_f^{(2)}(x_1, x_2, x_3, b_1, b_2, b_2) \} , \tag{32}
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_{a6}^{P(q)} &= 32\pi C_F \sqrt{2N_c} M_B^2 \int_0^1 [dx] \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \\
&\quad \times \{ [-r_K x_3 \phi_K(1 - x_2) \phi'_K(1 - x_3) - r_K (x_1 - x_2) \phi'_K(1 - x_2) \phi_K(1 - x_3)] \\
&\quad \times E_{a6}^{(q)'}(t_f^{(1)}) h_f^{(1)}(x_1, x_2, x_3, b_1, b_2, b_2) \\
&\quad - [r_K (2 - x_3) \phi_K(1 - x_2) \phi'_K(1 - x_3) - r_K (2 - x_1 - x_2) \phi'_K(1 - x_2) \phi_K(1 - x_3)] \\
&\quad \times E_{a6}^{(q)'}(t_f^{(2)}) h_f^{(2)}(x_1, x_2, x_3, b_1, b_2, b_2) \} , \tag{33}
\end{aligned}$$

with the definition $[dx] \equiv dx_1 dx_2 dx_3$ and $q = u$ and d .

The nonfactorizable amplitudes $\mathcal{M}_{a35}^{P(q)}$ are written as

$$\begin{aligned}
\mathcal{M}_{a35}^{P(q)} &= \mathcal{M}_{a3}^{P(q)} + \mathcal{M}_{a5}^{P(q)} , \\
\mathcal{M}_{a5}^{P(q)} &= -32\pi C_F \sqrt{2N_c} M_B^2 \int_0^1 [dx] \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \\
&\quad \times \{ [(x_1 - x_2) \phi_K(1 - x_2) \phi_K(1 - x_3) + r_K^2 (x_1 - x_2 - x_3) \phi'_K(1 - x_2) \phi'_K(1 - x_3)] \\
&\quad \times E_{a5}^{(q)'}(t_f^{(1)}) h_f^{(1)}(x_i, b_i) \\
&\quad + [x_3 \phi_K(1 - x_2) \phi_K(1 - x_3) + r_K^2 (2 + x_1 + x_2 + x_3) \phi'_K(1 - x_2) \phi'_K(1 - x_3)] \\
&\quad \times E_{a5}^{(q)'}(t_f^{(2)}) h_f^{(2)}(x_i, b_i) \} , \tag{34}
\end{aligned}$$

for $q = u$ and d . The evolution factors are given by

$$E_{ei}^{(s)'}(t) = \alpha_s(t) a_i^{(s)'}(t) \exp[-S(t)|_{b_3=b_1}] , \tag{35}$$

$$E_{ai}^{(q)'}(t) = \alpha_s(t) a_i^{(q)'}(t) \exp[-S(t)|_{b_3=b_2}] . \tag{36}$$

The expressions of \mathcal{M}_{a1} , \mathcal{M}_{a2} and $\mathcal{M}_{a3}^{P(q)}$ are the same as $\mathcal{M}_{a4}^{P(q)}$ but with the Wilson coefficients $a'_1(t_f)$, $a'_2(t_f)$, and $a'_3(t_f)$, respectively. The expressions of $\mathcal{M}_{a35}^{P(s)}$ are the same as $\mathcal{M}_{a35}^{P(q)}$ but with the kaon wave functions $\phi_K^{(i)}(1 - x_i)$ replaced by $\phi_K^{(i)}(x_i)$, $i = 2$ and 3 , and with the q quark replaced by the s quark. Notice the difference between the hard parts of $\mathcal{M}_{a6}^{P(q)}$ and $\mathcal{M}_{a5}^{P(q)}$, which are associated with the O_{5-8} operators. The former (latter) corresponds to the

fermion flow from the b quark to the d quark in the kaon (the \bar{d} quark in the B meson), *i.e.*, the case with one fermion loop (two fermion loops). That is, the nonfactorizable contributions associated with the structure $(V - A)(V + A)$ distinguish these two cases, while those associated with the structure $(V - A)(V - A)$ do not.

The functions $h^{(j)}$, $j = 1$ and 2 , appearing in Eqs. (30)-(33), are written as

$$\begin{aligned} h_d^{(j)} = & [\theta(b_1 - b_2) K_0(DM_B b_1) I_0(DM_B b_2) \\ & + \theta(b_2 - b_1) K_0(DM_B b_2) I_0(DM_B b_1)] \\ & \times K_0(D_j M_B b_2), \quad \text{for } D_j^2 \geq 0, \\ & \times \frac{i\pi}{2} H_0^{(1)}(\sqrt{|D_j^2|} M_B b_2), \quad \text{for } D_j^2 \leq 0, \end{aligned} \quad (37)$$

$$\begin{aligned} h_f^{(j)} = & \frac{i\pi}{2} \left[\theta(b_1 - b_2) H_0^{(1)}(FM_B b_1) J_0(FM_B b_2) \right. \\ & \left. + \theta(b_2 - b_1) H_0^{(1)}(FM_B b_2) J_0(FM_B b_1) \right] \\ & \times K_0(F_j M_B b_1), \quad \text{for } F_j^2 \geq 0, \\ & \times \frac{i\pi}{2} H_0^{(1)}(\sqrt{|F_j^2|} M_B b_1), \quad \text{for } F_j^2 \leq 0, \end{aligned} \quad (38)$$

with the variables

$$\begin{aligned} D^2 &= x_1 x_3, \\ D_1^2 &= F_1^2 = (x_1 - x_2) x_3, \\ D_2^2 &= -(1 - x_1 - x_2) x_3, \\ F^2 &= x_2 x_3, \\ F_2^2 &= x_1 + x_2 + (1 - x_1 - x_2) x_3. \end{aligned} \quad (39)$$

For details of the derivation of $h^{(j)}$, refer to [26]. The hard scales $t^{(j)}$ are chosen as

$$\begin{aligned} t_d^{(1)} &= \max(DM_B, \sqrt{|D_1^2|} M_B, 1/b_1, 1/b_2), \\ t_d^{(2)} &= \max(DM_B, \sqrt{|D_2^2|} M_B, 1/b_1, 1/b_2), \\ t_f^{(1)} &= \max(FM_B, \sqrt{|F_1^2|} M_B, 1/b_1, 1/b_2), \\ t_f^{(2)} &= \max(FM_B, \sqrt{|F_2^2|} M_B, 1/b_1, 1/b_2). \end{aligned} \quad (40)$$

In the above expressions the Wilson coefficients are defined by

$$\begin{aligned} a_1 &= C_2 + \frac{C_1}{N_c}, \\ a'_1 &= \frac{C_1}{N_c}, \\ a_2 &= C_1 + \frac{C_2}{N_c}, \\ a'_2 &= \frac{C_2}{N_c}, \\ a_3^{(q)} &= C_3 + \frac{C_4}{N_c} + \frac{3}{2} e_q \left(C_9 + \frac{C_{10}}{N_c} \right), \\ a_3^{(q)'} &= \frac{1}{N_c} \left(C_4 + \frac{3}{2} e_q C_{10} \right), \\ a_4^{(q)} &= C_4 + \frac{C_3}{N_c} + \frac{3}{2} e_q \left(C_{10} + \frac{C_9}{N_c} \right), \\ a_4^{(q)'} &= \frac{1}{N_c} \left(C_3 + \frac{3}{2} e_q C_9 \right), \end{aligned}$$

$$\begin{aligned}
a_5^{(q)} &= C_5 + \frac{C_6}{N_c} + \frac{3}{2}e_q \left(C_7 + \frac{C_8}{N_c} \right) , \\
a_5^{(q)'} &= \frac{1}{N_c} \left(C_6 + \frac{3}{2}e_q C_8 \right) , \\
a_6^{(q)} &= C_6 + \frac{C_5}{N_c} + \frac{3}{2}e_q \left(C_8 + \frac{C_7}{N_c} \right) , \\
a_6^{(q)'} &= \frac{1}{N_c} \left(C_5 + \frac{3}{2}e_q C_7 \right) .
\end{aligned} \tag{41}$$

Both QCD and electroweak penguin contributions have been included as shown in Eq. (41). It is expected that electroweak penguin contributions are small, as concluded in [32].

The pseudovector and pseudoscalar kaon wave functions ϕ_K and ϕ'_K are defined by

$$\phi_K(x) = \int \frac{dy^+}{2\pi} e^{-ixP_3^- y^+} \frac{1}{2} \langle 0 | \bar{u}(y^+) \gamma^- \gamma_5 s(0) | \pi \rangle , \tag{42}$$

$$\frac{m_{0K}}{P_3^-} \phi'_K(x) = \int \frac{dy^+}{2\pi} e^{-ixP_3^- y^+} \frac{1}{2} \langle 0 | \bar{u}(y^+) \gamma_5 s(0) | \pi \rangle , \tag{43}$$

respectively, satisfying the normalization

$$\int_0^1 dx \phi_K(x) = \int_0^1 dx \phi'_K(x) = \frac{f_K}{2\sqrt{2N_c}} . \tag{44}$$

The factor r_K ,

$$r_K = \frac{m_{0K}}{M_B} , \quad m_{0K} = \frac{M_K^2}{m_s + m_d} , \tag{45}$$

with m_s and m_d being the masses of the s and d quarks, respectively, is associated with the normalization of the pseudoscalar wave function ϕ'_K . Note that we have included the intrinsic b dependence for the heavy meson wave function ϕ_B but not for the kaon wave functions [7]. As the transverse extent b approaches zero, the B meson wave function $\phi_B(x, b)$ reduces to the standard parton model $\phi_B(x)$, *i.e.*, $\phi_B(x) = \phi_B(x, b = 0)$, which satisfies the normalization

$$\int_0^1 \phi_B(x) dx = \frac{f_B}{2\sqrt{2N_c}} . \tag{46}$$

V. NUMERICAL ANALYSIS

In the factorization formulas derived in Sec. IV, the Wilson coefficients evolve with the hard scale t that depends on the internal kinematic variables x_i and b_i . Wilson coefficients at a scale $\mu < M_W$ are related to the corresponding ones at $\mu = M_W$ through usual RG equations. Since the typical scale t of a hard amplitude is smaller than the b quark mass $m_b = 4.8$ GeV, we further evolve the Wilson coefficients from $\mu = m_b$ down to $\mu = t$. For the scale t below the c quark mass $m_c = 1.5$ GeV, we still employ the evolution function with $f = 4$, instead of with $f = 3$, for simplicity, since the matching at m_c is less essential. Therefore, we set $f = 4$ in the RG evolution between t and $1/b$ governed by the quark anomalous dimension γ . The explicit expressions of $C_i(\mu)$ are referred to [7].

For the B meson wave function, we adopt the model [7]

$$\phi_B(x, b) = N_B x^2 (1-x)^2 \exp \left[-\frac{1}{2} \left(\frac{x M_B}{\omega_B} \right)^2 - \frac{\omega_B^2 b^2}{2} \right] , \tag{47}$$

with the shape parameter $\omega_B = 0.4$ GeV [33]. The normalization constant $N_B = 91.7835$ GeV is related to the decay constant $f_B = 190$ MeV. The kaon wave functions are chosen as

$$\phi_K(x) = \frac{3}{\sqrt{2N_c}} f_K x(1-x)[1 + 0.51(1-2x) + 0.3(5(1-2x)^2 - 1)] , \quad (48)$$

$$\phi'_K(x) = \frac{3}{\sqrt{2N_c}} f_K x(1-x) . \quad (49)$$

ϕ_K is derived from QCD sum rules [34], where the second term $1 - 2x$, rendering ϕ_K a bit asymmetric, corresponds to $SU(3)$ symmetry breaking effect. The decay constant f_K is set to 160 MeV (in the convention $f_\pi = 130$ MeV). The wave functions ϕ_B and ϕ'_K were determined from the data of the $B \rightarrow K\pi$ decays [7].

We employ $G_F = 1.16639 \times 10^{-5} \text{ GeV}^{-2}$, the Wolfenstein parameters $\lambda = 0.2196$, $A = 0.819$, and $R_b = 0.38$, the unitarity angle $\phi_3 = 90^\circ$, the masses $M_B = 5.28 \text{ GeV}$, $M_K = 0.49 \text{ GeV}$, and $m_s = 100 \text{ MeV}$, which correspond to $m_{0K} = 2.22 \text{ GeV}$ [7], and the \bar{B}_d^0 (B^-) meson lifetime $\tau_{B^0} = 1.55 \text{ ps}$ ($\tau_{B^-} = 1.65 \text{ ps}$) [28]. Our predictions for the branching ratio of each mode are

$$\begin{aligned} B(B^+ \rightarrow K^+ K^0) &= 1.47 \times 10^{-6} , \\ B(B^- \rightarrow K^- K^0) &= 1.84 \times 10^{-6} , \\ B(B_d^0 \rightarrow K^+ K^-) &= 3.27 \times 10^{-8} , \\ B(\bar{B}_d^0 \rightarrow K^- K^+) &= 5.90 \times 10^{-8} , \\ B(B_d^0 \rightarrow K^0 \bar{K}^0) &= 1.75 \times 10^{-6} , \\ B(\bar{B}_d^0 \rightarrow K^0 \bar{K}^0) &= 1.75 \times 10^{-6} . \end{aligned} \quad (50)$$

The above values are lower than those of the $B \rightarrow \pi\pi$ decays [8,9]. Since the $B_d^0 \rightarrow K^\pm K^\mp$ modes involved only nonfactorizable annihilation amplitudes, their branching ratios are much smaller than those of the $B^\pm \rightarrow K^\pm K^0$ and $B_d^0 \rightarrow K^0 \bar{K}^0$ modes. As explained in Sec. II, a large deviation of future experimental data from the predicted $B_d^0 \rightarrow K^\pm K^\mp$ branching ratios will imply the existence of large FSI effects.

So far, CLEO gives only the upper bound of the $B \rightarrow KK$ decays [19]:

$$\begin{aligned} B(B^\pm \rightarrow K^\pm K^0) &< 5.1 \times 10^{-6} , \\ B(B_d^0 \rightarrow K^\pm K^\mp) &< 2.0 \times 10^{-6} . \end{aligned} \quad (51)$$

We also quote the upper bound

$$B(B_d^0 \rightarrow K^0 K^0) < 1.7 \times 10^{-5} , \quad (52)$$

from [28]. Obviously, our predictions are consistent with the above data.

The CP asymmetries are defined by

$$A_{CP} = \frac{B(\bar{B} \rightarrow KK) - B(B \rightarrow KK)}{B(\bar{B} \rightarrow KK) + B(B \rightarrow KK)} . \quad (53)$$

Employing the above set of parameters and $\phi_3 = 90^\circ$, we predict

$$\begin{aligned} A_{CP}(B^\pm \rightarrow K^\pm K^0) &= 0.11 , \\ A_{CP}(B_d^0 \rightarrow K^\pm K^\mp) &= 0.29 , \\ A_{CP}(B_d^0 \rightarrow K^0 K^0) &= 0 . \end{aligned} \quad (54)$$

Basically, the values are of the same order of those in the $B \rightarrow K\pi$ decays [7]. The CP asymmetry in the $K^0 \bar{K}^0$ modes vanishes, because they involve only penguin contributions. Measurements of the CP asymmetry in the $B_d^\pm \rightarrow K^\pm K^0$ can justify the PQCD evaluation of annihilation and nonfactorizable contributions to two-body B meson decays, and distinguish the FA, BBNS and PQCD approaches. The significant CP asymmetry observed in the $B_d^0 \rightarrow K^0 K^0$ will indicate strong FSI effects.

The dependences of the $B \rightarrow KK$ branching ratios on the angle ϕ_3 are displayed in Fig. 4. The branching ratios of the $K^\pm K^0$ modes increase with ϕ_3 , while those of the $K^\pm K^\mp$ modes decrease with ϕ_3 . The branching ratios of the $K^0 \bar{K}^0$ modes are insensitive to the variation of ϕ_3 . The variation with ϕ_3 is mainly a consequence of the interference between the penguin contributions and the nonfactorizable annihilation contributions \mathcal{M}_a from the tree operators. Since \mathcal{M}_{a1} in Eqs. (15) and (16) and \mathcal{M}_{a2} in Eqs. (17) and (18) contain the Wilson coefficients a'_1 and a'_2 , respectively, which are opposite in sign, the behaviors of the branching ratios with ϕ_3 in Figs. 4(a) and 4(b) are different.

The dependences of the CP asymmetries on the angle ϕ_3 are displayed in Fig. 5. The CP asymmetry in the $K^0 \bar{K}^0$ modes remains vanishing. The CP asymmetry in the $K^\pm K^\mp$ modes drop suddenly from 70% to zero near the high end of ϕ_3 . Since their branching ratios and the denominator in the definition of A_{CP} are small, the variation with ϕ is amplified. Figures 4 and 5 can be employed to determine the range of the angle ϕ_3 , when compared with future data.

VI. CONCLUSION

In this paper we have predicted the branching ratios and the CP asymmetries of all the $B \rightarrow KK$ modes using PQCD factorization theorem. The unitarity angle $\phi_3 = 90^\circ$ and the universal B and K meson wave functions extracted from the data of the $B \rightarrow K\pi$ and $\pi\pi$ decays have been employed. The dependences of the branching ratios and the CP asymmetries on the angle ϕ_3 have been also presented. These predictions can be confronted with future experimental data. We believe that these modes can be observed in B factories, which have started their operation recently.

The $B \rightarrow KK$ decays are very important for understanding dynamics of nonleptonic two-body B meson decays, such as FSI, annihilation and nonfactorizable effects. In the PQCD formalism FSI effects have been assumed to be small. As explained in Sec. II, the $B \rightarrow KK$ decays are more sensitive to these effects compared to the $B \rightarrow K\pi$ and $\pi\pi$ decays. Hence, the comparison of our predictions, especially for the CP asymmetry in the $B_d^0 \rightarrow K^0 K^0$ decays and for the $B_d^0 \rightarrow K^\pm K^\mp$ branching ratios, with future data provides a justification of the assumption. In PQCD the CP asymmetry of the $B^\pm \rightarrow K^\pm K^0$ modes depends on annihilation amplitudes. It has been argued that CP asymmetries in the $B \rightarrow KK$ decays are small in the FA and BBNS approaches, where annihilation contributions have been neglected. Therefore, experimental data of CP asymmetries will distinguish the FA, BBNS and PQCD approaches. The $B_d^0 \rightarrow K^\pm K^\mp$ modes involve only nonfactorizable annihilation amplitudes, such that their branching ratios can not be estimated in FA and BBNS. Future data of these modes can also verify the PQCD evaluation of the above contributions.

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modes	intermediate states	affected topologies	branching ratios for intermediate states [35,28]	data of branching ratios
$B^+ \rightarrow K^+ \bar{K}^0$	$\bar{D}^+ D^0$	P_c	$< 6.7 \times 10^{-3}$	$< 5.1 \times 10^{-6}$
$B_d^0 \rightarrow K^+ K^-$	$\pi^+ \pi^-$	T, P_u	$4.7^{+1.8}_{-1.5} \pm 0.6 \times 10^{-6}$	$< 2.0 \times 10^{-6}$
	$\pi^0 \pi^0$	T, P_u	$< 9.3 \times 10^{-6}$	
	$K^0 \bar{K}^0$	P_t	$< 1.7 \times 10^{-5}$	
$B_d^0 \rightarrow K^0 \bar{K}^0$	$D^+ D^-$	P_c	$< 5.9 \times 10^{-3}$	$< 1.7 \times 10^{-5}$
	$\pi^+ \pi^-$	P_u	$4.7^{+1.8}_{-1.5} \pm 0.6 \times 10^{-6}$	
	$\pi^0 \pi^0$	P_u	$< 9.3 \times 10^{-6}$	

TABLE I. FSI effects in the $B \rightarrow KK$ decays.

Figure Captions

1. Fig. 1: Feynman diagrams for the $B^\pm \rightarrow K^\pm K^0$ decays.
2. Fig. 2: Feynman diagrams for the $B_d^0 \rightarrow K^\pm K^\mp$ decays.
3. Fig. 3: Feynman diagrams for the $B_d^0 \rightarrow K^0 \bar{K}^0$ decays.
4. Fig. 4: Dependences of the branching ratios on ϕ_3 for (a) the $B^\pm \rightarrow K^\pm K^0$ modes, (b) the $B_d^0 \rightarrow K^\pm K^\mp$ modes and (c) the $B_d^0 \rightarrow K^0 \bar{K}^0$ modes. The upper (lower) lines correspond to the \bar{B} (B) meson decays.
5. Fig. 5: Dependence of CP asymmetries on ϕ_3 for (a) the $B^\pm \rightarrow K^\pm K^0$ modes, (b) the $B_d^0 \rightarrow K^\pm K^\mp$ modes and (c) the $B_d^0 \rightarrow K^0 \bar{K}^0$ modes.

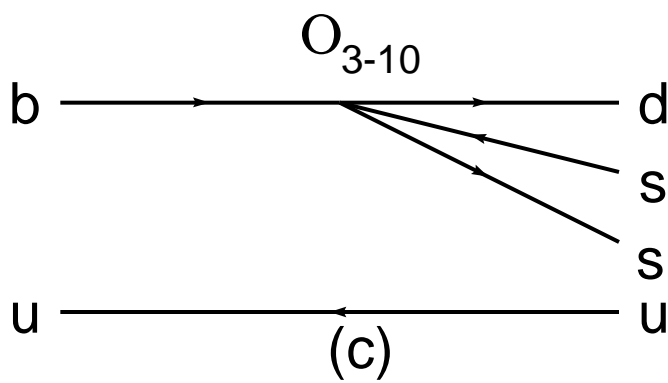
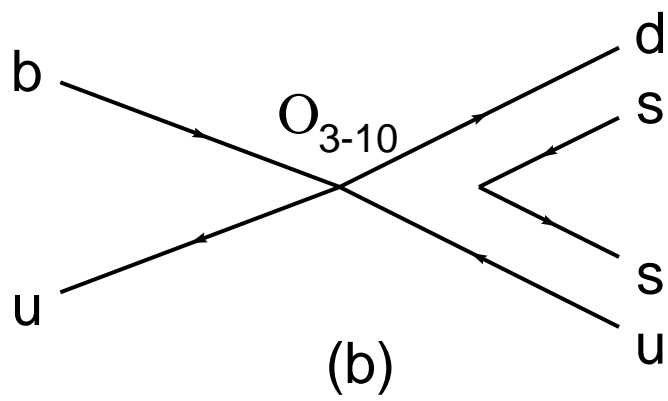
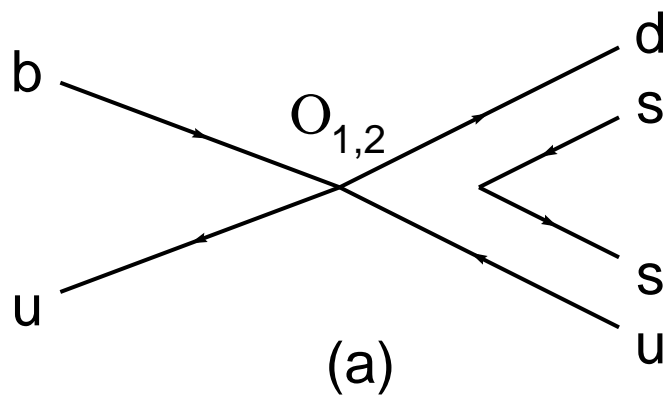


Fig.1

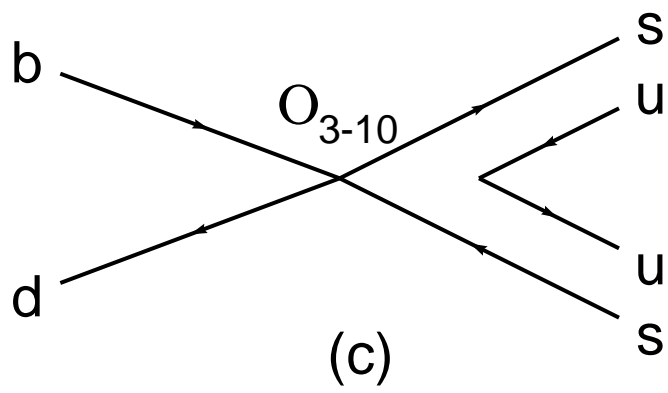
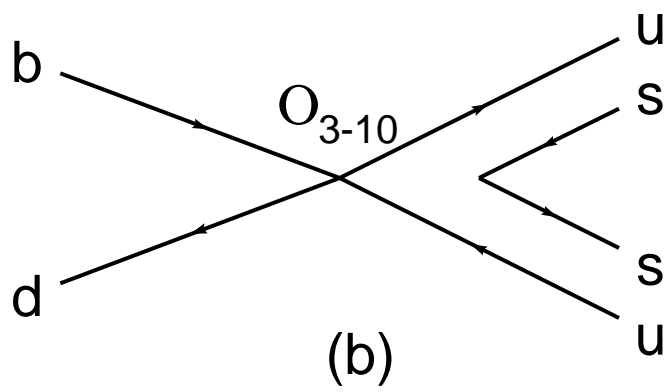
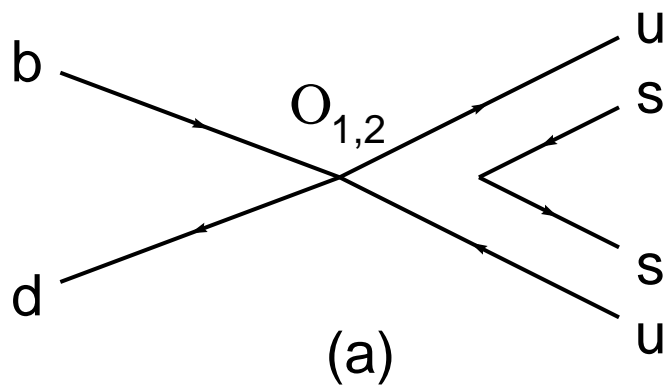


Fig.2

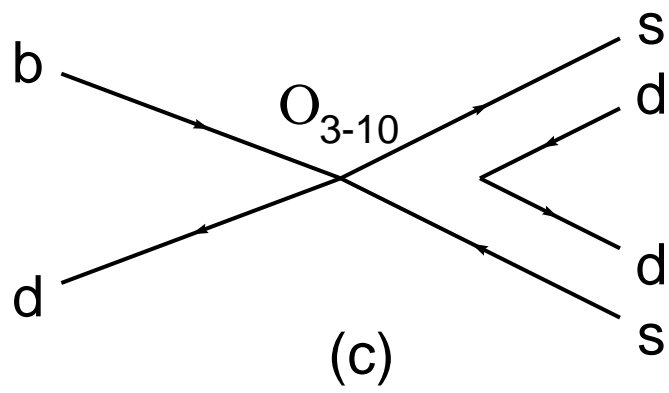
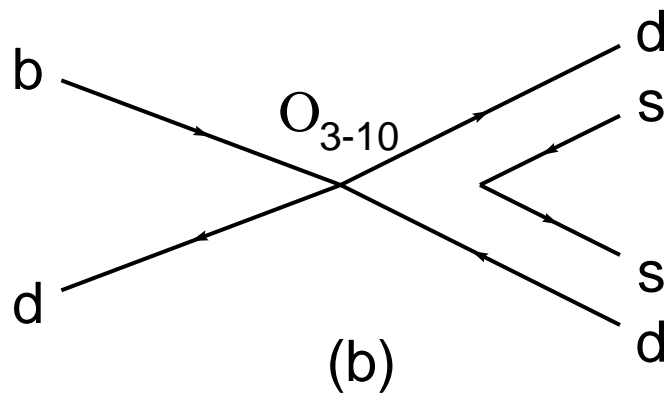
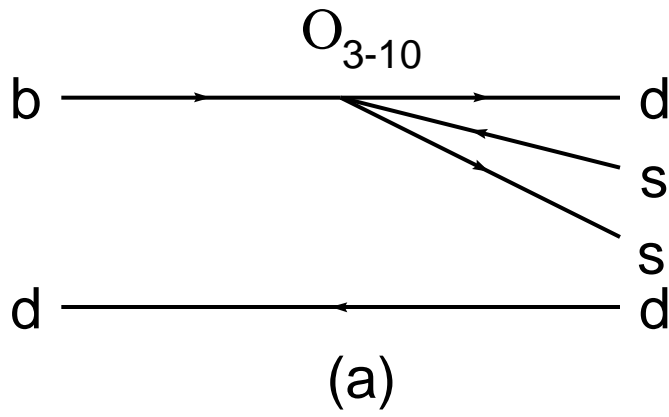


Fig.3

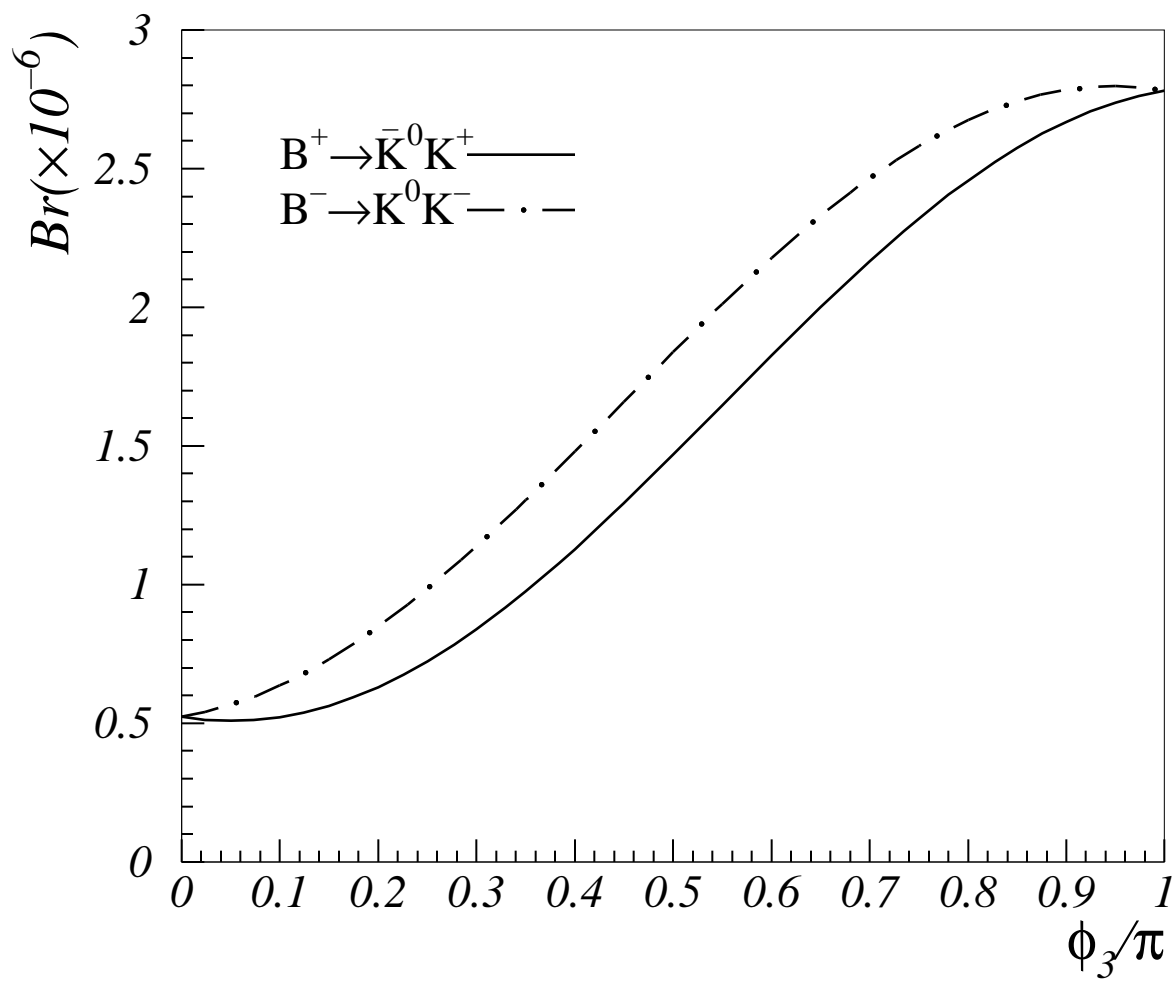


Fig.4a

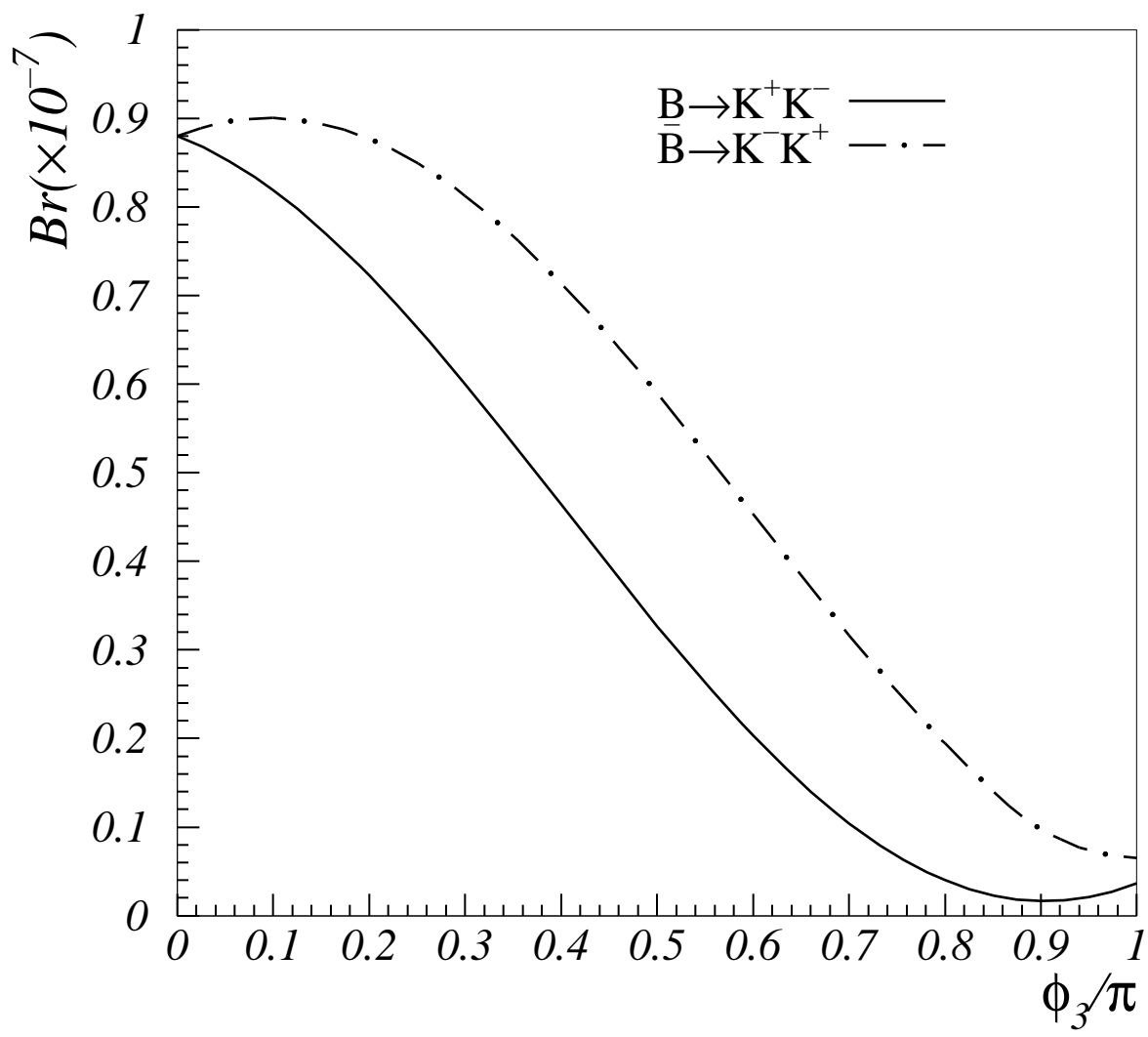


Fig.4b

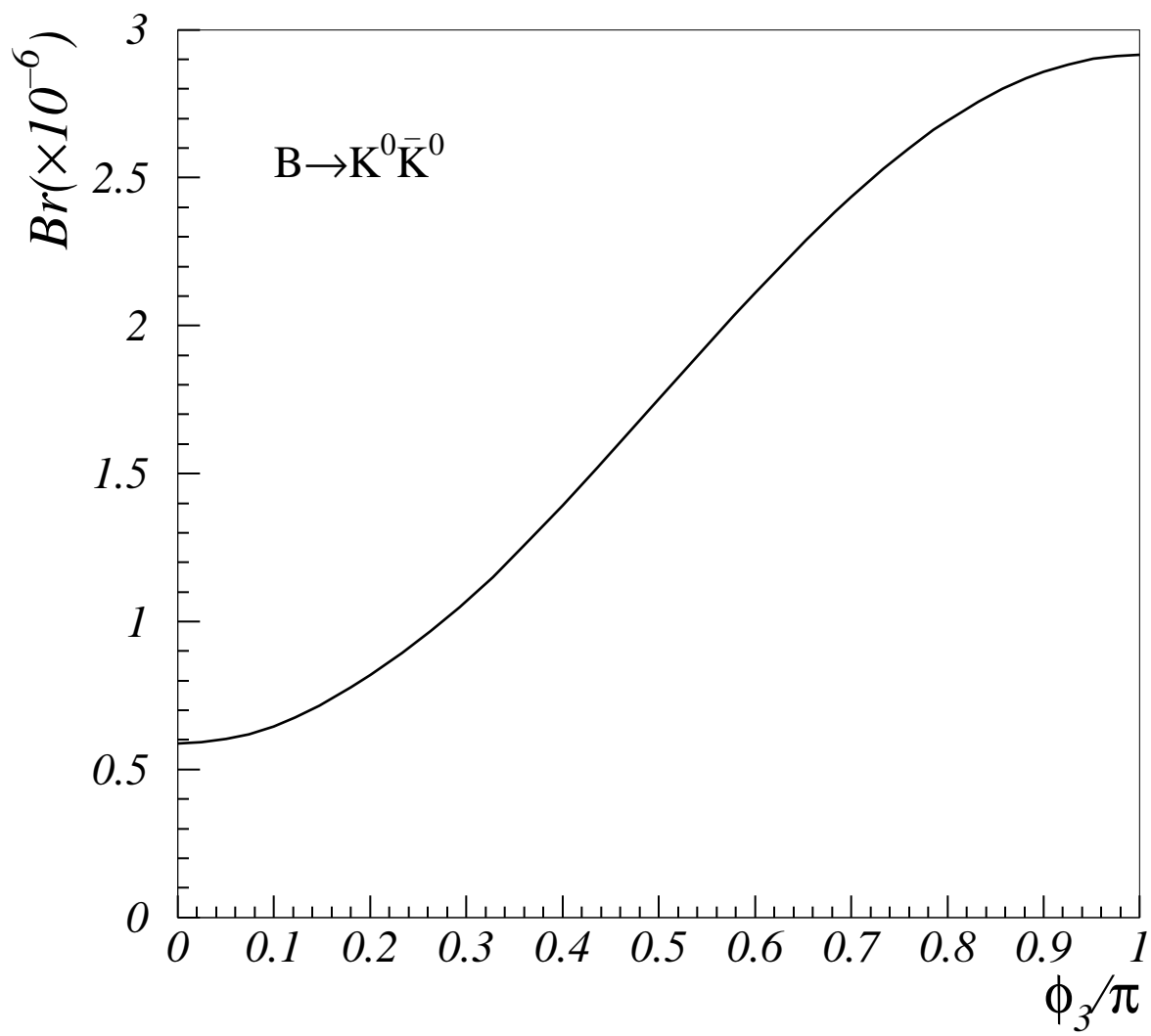


Fig.4c

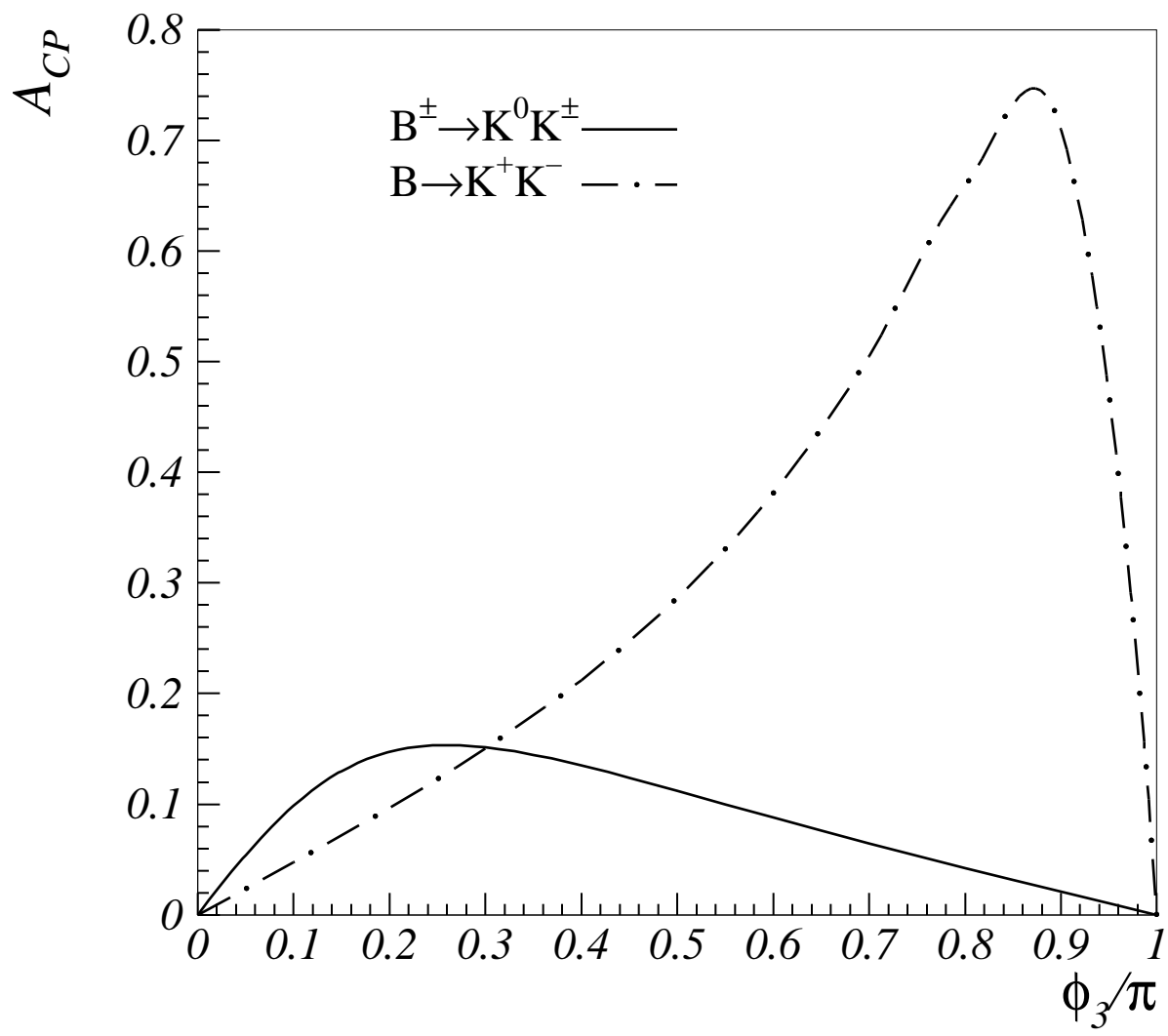


Fig.5